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# SYNCHRONIZATION OF FINITE-DIMENSIONAL "FORCE" GENERATORS

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The problem of synchronizing almost conservative dynamic objects [1] under weak constraints is considered. The character and mechanism of action of the objects on the supporting system are defined qualitatively and are no way related to its specific form [2].

The proposed procedure for investigating synchronous states in such systems is based on the notion of the dynamic influence matrix. It is shown that qualitative definition (specification) of the character of action of the objects is a natural basis for their classification. The paper ends with an examination of the synchronization of generators of "forces" of the simplest type, i.e. of one- and two-dimensional "forces".

The results can be applied, for example, to the solution of vibration engineering problems involving the properties of several complex vibration sources operating simultaneously.

The problems of synchronous state stability have already been investigated in [3 and 4], and therefore will not be considered here.

**1. The dynamic influence matrix.** Let us assume that the motion of the arbitrary $i$th object $(i = 1..., n)$ in a system is completely defined if we know the time variation of $l_i \times 1$ vector columns of its proper coordinates $q_i = (q_i^{(1)}, \ldots, q_i^{(l_i)})$ and $m_i \times 1$ vectors of the reverse influence parameters $\mathbf{x}_i = (x_i^{(1)}, \ldots, x_i^{(m_i)})$. The physical character of the reverse influence parameters is completely determined by the specifies of the object and is unrelated to the form of the supporting system [2]. The additional kinetic and additional potential energy of the object [1] can be naturally written in a form invariant relative to the form of the supporting system.

$$\Delta T_i = \mathbf{x}_i^{*\cdot} \mathbf{A}_{m_i}(\mathbf{x}_i, \mathbf{q}_i)\, \mathbf{q}_i^{\cdot} + {}^1/_2 \mathbf{x}_i^{*\cdot} \mathbf{A}_m(\mathbf{x}_i, \mathbf{q}_i)\, \mathbf{x}_i^{\cdot}$$

$$\Delta \Pi_i = \Delta \Pi_i'(\mathbf{x}_i, \mathbf{q}_i) \qquad (1.1)$$

Here $\mathbf{A}_{m_i}$ and $\mathbf{A}_m$ are, respectively, $m_i \times l_i$ and $m_i \times m_i$ matrices with not small coefficients dependent on $\mathbf{x}_i$ and $\mathbf{q}_i$ ; the asterisk denotes matrix transposition. On the other hand, the time variation of the reverse influence parameters is determined exclusively by the variation of the components of $m \times 1$ vectors $\mathbf{y} = (y_1, \ldots, y_m)$ of the absolute coordinates of the system, so that

$$\mathbf{x}_i = \mathbf{D}_i(\mathbf{y}) \qquad (1.2)$$

We note that the choices of the vectors $\mathbf{y}$ and $\mathbf{x}_i'(i = 1, \ldots \ldots, n)$ are in general completely unrelated. In practice the vector of reverse influence parameters often has a clear geometric significance, while the components of the vector $\mathbf{y}$ are sometimes fairly abstract (e.g. in the case of a supporting system with distributed parameters, when $m = \infty$).

Since the oscillations of the supporting system are small [1], so that

$$\mathbf{y} = \mu \mathbf{v} \qquad (1.3)$$

where $\mu$ is the coupling parameter, we have the expansion

$$\mathbf{x}_i = \mu \mathbf{u}_i + \mu^2 \cdots \qquad (\mathbf{u}_i = \mathbf{D}_{im}\mathbf{v}), \qquad \mathbf{D}_{im} = d\mathbf{D}_i(\mathbf{y}) / d\mathbf{y}\,|_{\mathbf{y}=0} \qquad (1.4)$$

Here $D_{im}$ is an $m_i \times m$ matrix with constant coefficients.

Relations (1.1) with allowance for (1.3) can be rewritten as

$$\Delta T_i = \mu \mathbf{u}_i^{*\cdot} \mathbf{A}_{mi}(0, \mathbf{q}_i)\, \mathbf{q}_i^{\cdot} + \mu^2 \ldots, \qquad \Delta \Pi_i = \mu \mathbf{u}_i^* \mathbf{C}_i(\mathbf{q}_i) + \mu^2 \cdots \qquad (1.5)$$

so that the additional kinetic potential of the $i$th object to within quantities of a higher order of smallness is

$$\mu \Delta L_i = \mu\, [u_i^{*\cdot} \mathbf{A}_{mi}(0, \mathbf{q}_i)\, \mathbf{q}_i^{\cdot} - \mathbf{u}_i^* \mathbf{C}_i(\mathbf{q}_i)] \qquad (1.6)$$

The potential generalized force conveyed to the supporting system by the $i$th object is given with the same degree of accuracy by

$$\left(\frac{d}{dt}\frac{\partial}{\partial \mathbf{y}^{\cdot}} - \frac{\partial}{\partial \mathbf{y}}\right)\mu \Delta L_i = -\mathbf{D}_{mi}\mathbf{F}_i$$

Here $\mathbf{D}_{mi} = \mathbf{D}_{im}^*$ and the $m_i \times 1$ vector

$$\mathbf{F}_i = -\left(\frac{d}{dt}\frac{\partial}{\partial \mathbf{u}_i^{\cdot}} - \frac{\partial}{\partial \mathbf{u}_i}\right)\Delta L_i = -\frac{d}{dt}[\mathbf{A}_{mi}(0, \mathbf{q}_i)\, \mathbf{q}_i^{\cdot}] + \mathbf{C}_i(\mathbf{q}_i) \qquad (1.7)$$

is the force due to the $i$th object reduced to its proper reverse influence parameters, i.e. the vector of fixed actions of the object on the supporting system. Thus, $\mathbf{F}_i$ is the physical "force" generated by the $i$th object, and the matrix $\mathbf{D}_{mi}$ characterizes the distribution of actions of this force over the supporting system. If the vector of the reverse influence parameters has three orthogonal spatial components, then the vector $\mathbf{F}_i$ is a force in the ordinary mechanical sense of the word ; but if the reverse influence parameters are rotations, then $\mathbf{F}_i$ varies periodically with time and is a function of the proper rapidly rotating phase,

$$\mathbf{F}_i = \mathbf{F}_i(\tau + \alpha_i, \nu) \qquad (\tau = \nu t) \qquad (1.8)$$

where $\alpha_i$ is the phase shift of the synchronous motion of the $i$th object.

The differential equation of small oscillations of the supporting system in the generating approximation is

$$\mathbf{M}\mathbf{v}^{\cdot\cdot} + \mathbf{B}\mathbf{v}^{\cdot} + \mathbf{C}\mathbf{v} = \sum_{i=1}^{n} \mathbf{D}_{mi}\mathbf{F}_i(\tau + \alpha_i, \nu) \qquad (1.9)$$

where $\mathbf{M}$ and $\mathbf{C}$ are symmetric positively defined $m \times m$ matrices with constant coefficients. The symmetric part of the $m \times m$ matrix $\mathbf{B}$ is also positive. Let us introduce the impulsive periodic function $\Phi(\tau)$ representable as a generalized Fourier series,

$$\Phi(\tau) = \frac{1}{2\pi} + \frac{1}{\pi} \sum_{s=1}^{\infty} \cos s\tau \tag{1.10}$$

The periodic solution of system (1.1) can be represented as a contraction [5],

$$\mathbf{v} = \sum_{i=1}^{n} \int_{0}^{2\pi} \Psi_m(\tau - \eta, \nu)\, \mathbf{D}_{mi} \mathbf{F}_i(\eta + \alpha_i, \nu)\, d\eta \tag{1.11}$$

where the $m \times m$ matrix $\Psi_m$ is the pulse-frequency characteristic of the supporting system which satisfies Eq.

$$\nu^2 \mathbf{M} \Psi_m'' + \nu \mathbf{B} \Psi_m' + \mathbf{C} \Psi_m = \mathbf{E}_m \Phi(\tau) \tag{1.12}$$

Here $\mathbf{E}_m$ is a unit $m \times m$ matrix, and the prime denotes differentiation with respect to $\tau$.

The vector of reverse influence parameters is given (in accordance with (1.4)) by Formula

$$\mathbf{u}_i = \sum_{j=1}^{n} \int_{0}^{2\pi} \mathbf{K}_{ij}(\tau - \eta, \nu)\, \mathbf{F}_j(\eta + \alpha_j, \nu)\, d\eta \tag{1.13}$$

Here the $m_i \times m_j$ matrix

$$\mathbf{K}_{ij} = \mathbf{D}_{im} \Psi_m \mathbf{D}_{mj} \tag{1.14}$$

is the dynamic matrix of influence of an arbitrary $j$th object on the $i$th object.

Now let us assume that the system of forces exerted by the objects is of the form

$$\mathbf{F}_j = \mathbf{E}_{m_j} \Phi(\tau)\, \delta_{lj} \tag{1.15}$$

where $l$ is a fixed natural number $(1 \leqslant l \leqslant n)$.

Then, by virtue of (1.13), we have

$$\mathbf{u}_i^{(l)} = \mathbf{K}_{il}(\tau, \nu) \tag{1.16}$$

Thus, the component $\varkappa_{ij}^{(p,q)}$ $(p = 1, \ldots, m_i;\ q = 1, \ldots, m_j)$ of the matrix $\mathbf{K}_{ij}$ yields the law of variation of the $p$th reverse influence parameter of the $i$th object on the periodic impulsive perturbation $\Phi(\tau)$ from the $q$th output of the $j$th object.

If small oscillations of the supporting system are not accompanied by the action of gyroscopic forces, so that $\mathbf{B} = \mathbf{B}^*$, then the matrix pulse-frequency characteristic $\Psi_m$ of the supporting system is also symmetric. Because of this we have the conditions of dynamic reciprocity

$$\mathbf{K}_{ij} = \mathbf{K}_{ji}^* \tag{1.17}$$

Finally, we note that by (1.16) the Fourier coefficients of the expansion of the dynamic influence matrix

$$\mathbf{K}_{ij}(\tau, \nu) = {}^1/_2\pi \mathbf{K}_{ij}^{(0)} + \frac{1}{\pi} \sum_{s=1}^{\infty} (\mathbf{K}_{ij}^{(s)} \cos s\tau + \mathbf{K}_{*ij}^{(s)} \sin s\tau) \tag{1.18}$$

are matrix harmonic influence coefficients. If there is no friction in the supporting system, then, of course, $K_{*ij}^{(s)} = 0$.

**2. The conditions of existence of a synchronous mode of a system.** It is shown in [4] that the generating configuration of phase shifts of synchronous motions of almost conservative objects under supporting constraints can be determined from the system

$$P_i(\alpha_1, \ldots, \alpha_n) \equiv \frac{1}{k_i(\nu)} \left[ f_i(\nu) - \sum_{j=1}^{n} W_{ij} \right] \tag{2.1}$$

Here $k_i(\nu)$ is the slope of the skeleton curve of the $i$th object, and $f_i(\nu)$ is the motion of its nonpotential forces in the generating approximation averaged over the period of synchronous motion. The partial vibrational moments $W_{ij}$ can be determined from Formulas

$$W_{ij} = 2\Phi_{ij} - \frac{1}{2}\left(\frac{\partial \Delta\Lambda_{ij}}{\partial \alpha_i} - \frac{\partial \Delta\Lambda_{ji}}{\partial \alpha_j}\right) \tag{2.2}$$

The possibility of introducing the quantities $\Delta\Lambda_{ij}$ and $\Phi_{ij}$ is due to the fact that small oscillations of the supporting system can be represented as sum (1.12), each of whose terms

$$v_i(\tau + \alpha_i, \nu) = \int_0^{2\pi} \Psi_m(\tau - \eta, \nu)\, D_{mi} F_i(\eta + \alpha_i, \nu)\, d\eta \tag{2.3}$$

characterizes the contribution of the corresponding object.

The equations for determing the components $v_i$ can be written as

$$\mathbf{M}\mathbf{v}_i^{\cdot\cdot} + \beta \mathbf{v}_i^{\cdot} + \mathbf{C}\mathbf{v}_i = -\left(\frac{d}{dt}\frac{\partial}{\partial \mathbf{v}_j^{\cdot}} - \frac{\partial}{\partial \mathbf{v}_j}\right)\Delta L_{ij} \tag{2.4}$$

where $\Delta L_{ij}$ is the additional kinetic potential of the $i$ th object due to the motion of the $j$ th object in the generating approximation,

$$\Delta L_{ij} = \mathbf{v}_j^{*\cdot} \mathbf{D}_{im} \mathbf{A}_{mi}(0, \mathbf{q}_i)\, \mathbf{q}_i^{\cdot} - \mathbf{v}_j^{*} \mathbf{D}_{im} \mathbf{C}_i(\mathbf{q}_i) \tag{2.5}$$

The energy characteristics of interaction of the $i$ th and $j$ th objects averaged over the period of synchronous motion, $\Delta\Lambda_{ij}$ and $\Phi_{ij}$, are given by

$$\Delta\Lambda_{ij}(\alpha_i - \alpha_j, \nu) = \frac{1}{2\pi}\int_0^{2\pi} \Delta L_{ij}\, d\tau$$

$$\Phi_{ij}(\alpha_i - \alpha_j, \nu) = \frac{1}{2\pi\nu}\int_0^{2\pi} \frac{1}{2} \mathbf{v}_i^{*\cdot} \mathbf{B}_c \mathbf{v}_i^{\cdot}\, d\tau \qquad (B_c = 1/2\,(B + B^*)) \tag{2.6}$$

It is not difficult to show that basic equations (2.1) remain unaltered in our more general case of objects with an arbitrary number of degrees of freedom.

Let us take the scalar product of matrix equation (2.4) and the vector row $\mathbf{v}_j^{*\cdot}$ and add the result to the analogous expression with the subscripts $i$ and $j$ interchanged. Let us then average the result over the period of synchronous motion.

Then, after some transformations we arrive directly at the relations

$$2\Phi_{ij} = -\frac{1}{2}\left(\frac{\partial \Delta\Lambda_{ij}}{\partial \alpha_i} + \frac{\partial \Delta\Lambda_{ji}}{\partial \alpha_j}\right) \qquad (i \neq j) \tag{2.7}$$

We can show that Formula (2.7) remains valid for $i = j$, if we assume that the quantities $q_i^{\cdot}$ and $q_i$ in the expression for $\Delta L_{ii}$ (2.5) are independent of the phase shifts $\alpha_i$. By virtue of this assumption we have

$$2\Phi_{ii} = -\frac{\partial \Delta\Lambda_{ii}}{\partial \alpha_i} \tag{2.8}$$

and the initial equations (2.1) for determining the generating phase shift configuration can be rewritten as

$$P_i \equiv \frac{1}{k_i(\nu)}\left[f_i(\nu) + \frac{\partial \Delta\Lambda_i}{\partial \alpha_i}\right] = 0, \qquad \Delta\Lambda_i = \frac{1}{2\pi}\int_0^{2\pi} \Delta L_i\, d\tau = \sum_{j=1}^{n} \Delta\Lambda_{ij} \tag{2.9}$$

It is natural to call the quantity $\Delta\Lambda_i$ in this expression the "additional action integral of the $i$ th object".

This is the general formulation of the conditions of existence of a synchronous mode which does not include the energy characteristics of motion of the supporting system.

Let us now turn to the determination of the additional action integral of the $i$ th object which can be obtained by averaging Expression (1.8). By some simple transformations involving integration by parts, allowing for (1.9) and (1.4), we obtain

$$\Delta\Lambda_i = \frac{1}{2\pi}\sum_{j=1}^{n}\int_0^{2\pi}\int_0^{2\pi} F_i^*(\tau+\alpha_i, \nu)\, K_{ij}(\tau-\eta, \nu)\, F_j(\eta+\alpha_j, \nu)\, d\tau\, d\eta \qquad (2.10)$$

By virtue of (1.12), relation (2.10) can be rewritten as

$$\Delta\Lambda_i = \frac{1}{2\pi}\int_0^{2\pi} \mathbf{F}_i^*(\tau+\alpha_i, \nu)\, \mathbf{u}_i(\tau, \nu)\, d\tau \qquad (2.11)$$

Thus, the additional action integral of the $i$th object is equal to the generalized force generated by the object (over its reactive vector displacement over the vector of reverse influence parameters) averaged over the period.

An expression for the partial vibrational moments (2.2) can be obtained by differentiating Expression (2.10) with respect to the phase shift $\alpha_i$ in accordance with (2.9). By virtue of assumption (2.8) we must assume here that the vectorial force $F_i(\eta+\alpha_i, \nu)$ in the $i$th term of sum (2.10) is independent of $\alpha_i$. Hence, integrating by parts the derivative of Expression (2.10) with respect to $\alpha_i$, we obtain

$$W_{ij}(\alpha_i-\alpha_j, \nu) = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi} \mathbf{F}_i^*(\tau+\alpha_i, \nu)\, \mathbf{K}_{ij}'(\tau-\eta, \nu)\, \mathbf{F}_j(\eta+\alpha_j, \nu)\, d\tau\, d\eta \qquad (2.12)$$

We note that in the absence of friction in the supporting system dynamic influence matrix (1.17) is an even function of time, $\mathbf{K}_{ij}(\tau, \nu) = \mathbf{K}_{ij}(-\tau, \nu)$, so that $\mathbf{K}_{ij}'(\tau, \nu) = -\mathbf{K}_{ij}'(-\tau, \nu)$.

Recalling dynamic reciprocity conditions (1.16), we readily infer from this that the matrix of partial vibrational moments is skew-symmetric,

$$W_{ij}(\alpha_i-\alpha_j, \nu) + W_{ji}(\alpha_j-\alpha_i, \nu) = 0 \qquad (2.13)$$

The above relation is at the same time a consequence of the integral criterion of synchronous motion stability [1].

**3. Synchronization of generators of forces of the simplest type.** The process of constructing a system of equations for determining the generating phase shift configuration can be broken down into three steps:

1) determination of the magnitudes and directions of the forces $\mathbf{F}_i$ exerted by the objects with a braked supporting system;

2) determination of the components of the dynamic influence matrix;

3) execution of simple matrix operations and integrations in accordance with Expression (2.12).

This enables us to classify synchronized objects according to the character and magnitude of the forces they generate. The most simple and at the same time one of the most common classes of objects of this type is clearly the class of generators of scalar influences, or less precisely, generators of forces of constant direction.

In this case the reverse influence parameter vector has only one component. For this reason all of the quantities appearing in relations (2.12) are scalar.

The harmonic coefficients in Expansion (1.19) are also scalar. Let

$$K_{ij}^{(s)} = k_{ij}^{(s)}\cos\psi_{ij}^{(s)} \qquad K_{ij*}^{(s)} = k_{ij}^{(s)}\sin\psi_{ij}^{(s)} \qquad (3.1)$$

where $k_{ij}^{(s)}$ and $\psi_{ij}^{(s)}$ are the influence coefficients of a unit harmonic force (in the generalized sense) of frequency $s$ in dimensionless time acting at the output of the $j$th rotation of the two vibrators is synchronized in some way, so that the sum inertial force

$$F(\tau+\alpha,\ \nu) = 2\ m\varepsilon\nu^2 \cos(\tau+\alpha)$$

has a constant direction when the supporting system is braked. The distance between the centers of rotation of the vibrators is small, and the point $O$ is attached directly to the supporting system.

Another example of the force generator of the same category is the crankgear mechanism (Fig. 2). We assume that the crank is balanced and has a large moment of inertia, that the connecting rod is weightless, that the axis of rotation and the generatirices are connected to the supporting system, and that the ratio $\varepsilon / l$ is of the order of smallness of the coupling parameter. The force generated by this mechanism has the constant direction $Ou$ in the generating approximation; it is conveyed to the supporting system through the point $O$ and is given by

$$F(\tau+\alpha,\ \nu) = m\varepsilon\nu^2\cos(\tau+\alpha)$$

2. A mechanical harmonic moment generator. The simplest generator of the harmonic moment of constant direction is the coupled mechanical vibrator described at the beginning of the preceding Section if the rotation of the constituent debalances is synchronized and cophased as shown in Fig. 3. The mechanical moment generated by the vibrator is perpendicular to the plane of the figure and is given by

$$F(\tau+\alpha,\ \nu) = 2m\varepsilon r\nu^2\cos(\tau+\alpha)$$

Now let us consider the class of two-dimensional force generators one step more complicated than those described above. Limiting ourselves to purely harmonic perturbations

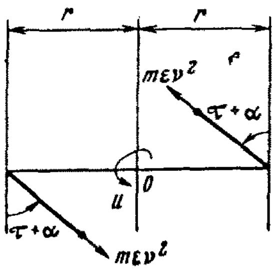


Fig. 3

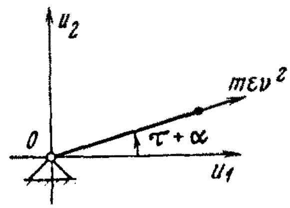


Fig. 4

by the objects, we choose their proper coordinates in such a way that the force at the first output is maximum at the instant when the rotation phase vanishes. Here, of course, we have

$$\mathrm{F}_i(\tau+\alpha_i,\ \nu) = \left\| \begin{matrix} F_i^{(1)}\cos(\tau+\alpha_i) \\ F_i^{(2)}\cos(\tau+\alpha_i-\gamma_i) \end{matrix} \right\| \qquad i=1,\ldots,n \tag{3.8}$$

The dynamic influence matrix of the $j$th object on the $i$th, or, more precisely, its purely harmonic part, can be written as

$$K_{ij}(\tau,\ \nu) = \frac{1}{\pi} \left\| \begin{matrix} k_{ij}^{(1,1)}\cos(\tau-\psi_{ij}^{(1,1)}) & k_{ij}^{(1,2)}\cos(\tau-\psi_{ij}^{(1,2)}) \\ k_{ij}^{(2,1)}\cos(\tau-\psi_{ij}^{(2,1)}) & k_{ij}^{(2,2)}\cos(\tau-\psi_{ij}^{(2,2)}) \end{matrix} \right\| \tag{3.9}$$

The coefficients $k_{ij}^{(p,q)}$ and $\psi_{ij}^{(p,q)}(p,\ q=1,\ 2)$ appearing in matrix (3.2) signify (in the mechanical case) the harmonic coefficients of the influence of the force (or moment, depending on the physical nature of the action) on the amplitude and phase of the displacement (or rotation).

Substitution of Expressions (3.8) and (3.9) into basic relation (2.12) yields the following general formula for the partial vibrational moment:

$$W_{ij}=\frac{1}{2}\left[F_i^{(1)}F_j^{(2)}k_{ij}^{(1.1)}\sin\left(\alpha_i-\alpha_j+\psi_{ij}^{(1.1)}\right)+F_i^{(1)}F_j^{(2)}k_{ij}^{(1.2)}\sin\left(\alpha_i-\alpha_j+\gamma_j+\varphi_{ij}^{(1.2)}\right)+\right.$$
$$\left.+F_i^{(2)}F_j^{(1)}k_{ij}^{(2.1)}\sin\left(\alpha_i-\alpha_j-\gamma_i+\psi_{ij}^{(2.1)}\right)+F_i^{(2)}F_j^{(2)}k_{ij}^{(2.2)}\sin\left(\alpha_i-\alpha_j-\gamma_i+\gamma_j+\psi_{ij}^{(2.2)}\right)\right] \tag{3.10}$$

The most common object of this class is a rotating force generator, i.e. an ordinary mechanical debalance vibrator (Fig. 4). The rotating vectorial force generated by a debalance generator is given by

$$\mathbf{F}(\tau+\alpha,\ \nu)=m\varepsilon\nu^2\begin{vmatrix}\cos(\tau+\alpha)\\ \sin(\tau+\alpha)\end{vmatrix} \tag{3.11}$$

Specialization of Expression (3.10) in accordance with this formula yields the expression for the partial vibrational moment of the problem of synchronizing mechanical vibrators mounted on a linear (more precisely, a linearizable) supporting system [7].

Some practical problems in vibration proofing involve the synchronization of generators of a helical "force" (dynamic screw). We are referring to a two-output dynamic object which generates a mechanical force and moment having the same constant direction with a braked supporting system.

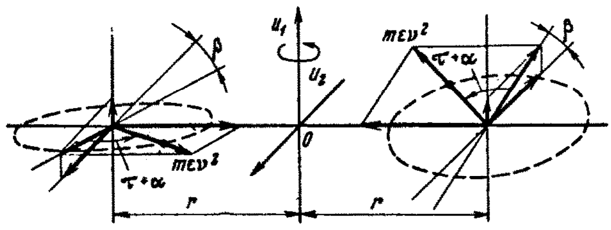


Fig. 5

It is clear that one of the components of the vector of reverse influence parameters of a dynamic screw generator is linear displacement along some axis, while the other component is rotation about the same axis. A dynamic screw generator can take the form of a coupled vibrator whose component flux rotations are synchronized and cophased as shown in Fig. 5. The object generates the helical "force"

$$\mathbf{F}(\tau+\alpha+\nu)=2m\varepsilon\nu^2\begin{vmatrix}\sin\beta\\ r\cos\beta\end{vmatrix}\cos(\tau+\alpha) \tag{3.12}$$

which is conveyed to the supporting system through the point $O$. For $\beta=1/2\,\pi$ this object degenerates into a harmonic force generator; for $\beta=0$ it degenerates into a moment generator.

We note in conclusion that the order of conversion of the components of the reverse influence parameter vector, and sometimes (e.g. in the case of a rotating force generator) the very choice of these components in problems involving synchronization of multidimensional force generators, are somewhat arbitrary and are generally dictated by purely physical considerations.

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# STATISTICAL MECHANICS OF GASEOUS SUSPENSIONS. DYNAMIC AND SPECTRAL EQUATIONS

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We present the model of random, pseudoturbulent phase motions in concentrated, disperse, gas-suspended particles systems, based on the development of ideas expounded in our previous papers [1 – 3] and [4]. This model enables us, in principle, to construct a structural theory of gaseous suspensions when the flow is pseudoturbulent [3], to compute the corresponding transfer coefficients and to formulate the dynamic equations of motion.

Papers [1 – 3] considered pulsating motions of a two-phase disperse system using the statistical approach and developed a general method of the quantitative treatment of the pulsations and their influence on the average motion of the system. At the same time, ways were indicated towards constructing a non-Newtonian mechanics of disperse systems.

The model [3] however, retains a number of unsolved difficulties. First of them concerns the fact that the proposed model is based on the use of certain random forces acting on the phases in random motion, and of the statistical white noise, the latter allowing the description of not only of the orderly degeneration of the fluctuations of the averaged hydrodynamic field of a disperse system, but also of their random accumulation. The forces and the white noise enter [3] separately, although the general physical considerations imply that a mere appearance of the white noise should be the result of the action of the random forces.